Foundation tier knowledge, skills and understanding

1. Number

Structure and calculation

What students need to learn:

N1 order positive and negative integers, decimals and fractions; use the symbols $=, \neq, <, >, \leq, \geq$

N2 apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)

N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

N4 use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem

N5 apply systematic listing strategies

N6 use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5

N7 calculate with roots, and with integer indices

N8 calculate exactly with fractions and multiples of π

N9 calculate with and interpret standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer

Fractions, decimals and percentages

What students need to learn:

N10 work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 or $\frac{3}{8}$)

N11 identify and work with fractions in ratio problems

N12 interpret fractions and percentages as operators

Measures and accuracy

What students need to learn:

N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate

N14 estimate answers; check calculations using approximation and estimation, including answers obtained using technology

N15 round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding

N16 apply and interpret limits of accuracy

2. Algebra

Notation, vocabulary and manipulation

What students need to learn:

A1 use and interpret algebraic manipulation, including:

- ab in place of $a \times b$
- $3y$ in place of $y + y + y$ and $3 \times y$
- a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a^2b in place of $a \times a \times b$
- $\frac{a}{b}$ in place of $a \div b$
- coefficients written as fractions rather than as decimals
- brackets

A2 substitute numerical values into formulae and expressions, including scientific formulae

A3 understand and use the concepts and vocabulary of expressions, equations, formulae, identities, inequalities, terms and factors

A4 simplify and manipulate algebraic expressions (including those involving surds) by:

- collecting like terms
- multiplying a single term over a bracket
- taking out common factors
- expanding products of two binomials
- factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares;
- simplifying expressions involving sums, products and powers, including the laws of indices

A5 understand and use standard mathematical formulae; rearrange formulae to change the subject

A6 know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments

A7 where appropriate, interpret simple expressions as functions with inputs and outputs.

Graphs

What students need to learn:

A8 work with coordinates in all four quadrants

A9 plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form $y = mx + c$ to identify parallel lines; find the equation of the line through two given points or through one point with a given gradient

A10 identify and interpret gradients and intercepts of linear functions graphically and algebraically

A11 identify and interpret roots, intercepts, turning points of quadratic functions graphically; deduce roots algebraically

A12 recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$

A14 plot and interpret graphs (including reciprocal graphs) and graphs of non-standard functions in real contexts to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

Solving equations and inequalities

What students need to learn:

A17 solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation); find approximate solutions using a graph

A18 solve quadratic equations algebraically by factorising; find approximate solutions using a graph

A19 solve two simultaneous equations in two variables (linear/linear algebraically; find approximate solutions using a graph

A21 translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution

A22 solve linear inequalities in one variable; represent the solution set on a number line

Sequences

What students need to learn:

A23 generate terms of a sequence from either a term-to-term or a position-to-term rule

A24 recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions (r^n where n is an integer, and r is a rational number > 0)

A25 deduce expressions to calculate the nth term of linear sequences

3. Ratio, proportion and rates of change

What students need to learn:

R1 change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts

R2 use scale factors, scale diagrams and maps

R3 express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1

R4 use ratio notation, including reduction to simplest form

R5 divide a given quantity into two parts in a given part:part or part:whole ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)

R6 express a multiplicative relationship between two quantities as a ratio or a fraction

R7 understand and use proportion as equality of ratios

R8 relate ratios to fractions and to linear functions

R9 define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease and original value problems, and simple interest including in financial mathematics

R10 solve problems involving direct and inverse proportion, including graphical and algebraic representations

R11 use compound units such as speed, rates of pay, unit pricing, density and pressure

R12 compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors

R13 understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$; interpret equations that describe direct and inverse proportion

R14 interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion

R16 set up, solve and interpret the answers in growth and decay problems, including compound interest

4. Geometry and measures

Properties and constructions

What students need to learn:

G1 use conventional terms and notation: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description

G2 use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line

G3 apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles; understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)

G4 derive and apply the properties and definitions of special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language

G5 use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)

G6 apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs

G7 identify, describe and construct congruent and similar shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement (including fractional scale factors)

G9 identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment

G11 solve geometrical problems on coordinate axes

G12 identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres

G13 construct and interpret plans and elevations of 3D shapes

Mensuration and calculation

What students need to learn:

G14 use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)

G15 measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings

G16 know and apply formulae to calculate: area of triangles, parallelograms, trapezia; volume of cuboids and other right prisms (including cylinders)

G17 know the formulae: circumference of a circle = $2\pi r = \pi d$, area of a circle = πr^2; calculate: perimeters of 2D shapes, including circles; areas of circles and composite shapes; surface area and volume of spheres, pyramids, cones and composite solids

G18 calculate arc lengths, angles and areas of sectors of circles

G19 apply the concepts of congruence and similarity, including the relationships between lengths, in similar figures

G20 know the formulae for: Pythagoras' theorem $a^2 + b^2 = c^2$, and the trigonometric ratios, $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ and $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$; apply them to find angles and lengths in right-angled triangles in two-dimensional figures

G21 know the exact values of $\sin\theta$ and $\cos\theta$ for $\theta = 0°, 30°, 45°, 60°$ and $90°$; know the exact value of $\tan\theta$ for $\theta = 0°, 30°, 45°$ and $60°$

Vectors

What students need to learn:

G24 describe translations as 2D vectors

G25 apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors

5. Probability

What students need to learn:

P1 record, describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees

P2 apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments

P3 relate relative expected frequencies to theoretical probability, using appropriate language and the 0-1 probability scale

P4 apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one

P5 understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size

P6 enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams and tree diagrams

P7 construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities

P8 calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions

6. Statistics

What students need to learn:

S1 infer properties of populations or distributions from a sample, while knowing the limitations of sampling

S2 interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time series data and know their appropriate use

S4 interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:

- appropriate graphical representation involving discrete, continuous and grouped data
- appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers)

S5 apply statistics to describe a population

S6 use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends while knowing the dangers of so doing