Higher tier knowledge, skills and understanding

1. Number

Structure and calculation

What students need to learn:

N1 order positive and negative integers, decimals and fractions; use the symbols $=, \neq, <, >, \leq, \geq$

N2 apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)

N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

N4 use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem

N5 apply systematic listing strategies, **including use of the product rule for counting (i.e. if there are *m* ways of doing one task and for each of these, there are *n* ways of doing another task, then the total number of ways the two tasks can be done is *m* × *n* ways)**

N6 use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5*;* **estimate powers and roots of any given positive number**

N7 <u>calculate with roots, and with integer</u> **and fractional** <u>indices</u>

N8 calculate exactly with fractions, **surds** <u>and multiples of π</u>*;* **simplify surd expressions involving squares (e.g. √12 = √(4 × 3) = √4 × √3 = 2√3) and rationalise denominators**

N9 calculate with and interpret standard form $A \times 10^{n}$, where $1 \leq A < 10$ and *n* is an integer

Fractions, decimals and percentages

What students need to learn:

N10   work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 or $\frac{3}{8}$); **change recurring decimals into their corresponding fractions and vice versa**

N11   identify and work with fractions in ratio problems

N12   interpret fractions and percentages as operators

Measures and accuracy

What students need to learn:

N13   use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate

N14   estimate answers; check calculations using approximation and estimation, including answers obtained using technology

N15   round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); <u>use inequality notation to specify simple error intervals due to truncation or rounding</u>

N16   <u>apply and interpret limits of accuracy</u>, **including upper and lower bounds**

2. Algebra

Notation, vocabulary and manipulation

What students need to learn:

A1   use and interpret algebraic manipulation, including:

- *ab* in place of $a \times b$

- 3*y* in place of $y + y + y$ and $3 \times y$

- a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a^2b in place of $a \times a \times b$

- $\frac{a}{b}$ in place of $a \div b$

- coefficients written as fractions rather than as decimals

- brackets

A2   substitute numerical values into formulae and expressions, including scientific formulae

A3   understand and use the concepts and vocabulary of expressions, equations, formulae, <u>identities</u>, inequalities, terms and factors

A4   simplify and manipulate algebraic expressions (<u>including those involving</u> <u>surds</u> **and algebraic fractions**) by:

- collecting like terms

- multiplying a single term over a bracket

- taking out common factors

- <u>expanding products of two</u> **or more** <u>binomials</u>

- <u>factorising quadratic expressions of the form $x^2 + bx + c$, including the</u> <u>difference of two squares</u>; **factorising quadratic expressions of the** **form $ax^2 + bx + c$**

- simplifying expressions involving sums, products and powers, including the laws of indices

A5   understand and use standard mathematical formulae; rearrange formulae to change the subject

A6   <u>know the difference between an equation and an identity; argue</u> <u>mathematically to show algebraic expressions are equivalent, and use</u> <u>algebra to support and construct arguments</u> **and proofs**

A7   where appropriate, interpret simple expressions as functions with inputs and outputs**; interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function' (the use of formal function notation is expected)**

Graphs

What students need to learn:

A8   work with coordinates in all four quadrants

A9   plot graphs of equations that correspond to straight-line graphs in the coordinate plane; <u>use the form $y = mx + c$ to identify parallel</u> **and** **perpendicular lines**; <u>find the equation of the line through two given points</u> <u>or through one point with a given gradient</u>

A10   identify and interpret gradients and intercepts of linear functions graphically and algebraically

A11   <u>identify and interpret roots, intercepts, turning points of quadratic functions</u> <u>graphically; deduce roots algebraically</u> **and turning points by completing the square**

A12   recognise, sketch and interpret graphs of linear functions, quadratic functions, <u>simple cubic functions, the reciprocal function</u> $y = \dfrac{1}{x}$ <u>with $x \neq 0$</u>,

exponential functions $y = k^x$ for positive values of k, and the **trigonometric functions (with arguments in degrees) $y = \sin x$,** **$y = \cos x$ and $y = \tan x$ for angles of any size**

A13   **sketch translations and reflections of a given function**

A14 plot and interpret graphs (<u>including reciprocal graphs</u> **and exponential graphs**) and graphs of non-standard functions in real contexts to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

A15 **calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts (this does not include calculus)**

A16 **recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point**

Solving equations and inequalities

What students need to learn:

A17 solve linear equations in one unknown algebraically <u>(including those with the unknown on both sides of the equation)</u>; find approximate solutions using a graph

A18 <u>solve quadratic equations</u> (**including those that require rearrangement**) <u>algebraically by factorising</u>, **by completing the square and by using the quadratic formula**; <u>find approximate solutions using a graph</u>

A19 <u>solve two simultaneous equations in two variables (linear/linear **or** linear/quadratic</u>) <u>algebraically</u>; <u>find approximate solutions using a graph</u>

A20 **find approximate solutions to equations numerically using iteration**

A21 <u>translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution</u>

A22 <u>solve linear inequalities in one</u> **or two** <u>variable</u>**(s)**, **and quadratic inequalities in one variable;** <u>represent the solution set on a number line</u>, **using set notation and on a graph**

Sequences

What students need to learn:

A23 generate terms of a sequence from either a term-to-term or a position-to-term rule

A24 recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, <u>Fibonacci type sequences, quadratic sequences, and simple geometric progressions (r^n where n is an integer, and r is a rational number > 0</u> **or a surd**) **and other sequences**

A25 deduce expressions to calculate the nth term of linear **and quadratic** sequences

3. Ratio, proportion and rates of change

What students need to learn:

R1   change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, <u>density, pressure</u>) in numerical <u>and algebraic</u> contexts

R2   use scale factors, scale diagrams and maps

R3   express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1

R4   use ratio notation, including reduction to simplest form

R5   divide a given quantity into two parts in a given part:part or part:whole ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)

R6   express a multiplicative relationship between two quantities as a ratio or a fraction

R7   understand and use proportion as equality of ratios

R8   relate ratios to fractions and to linear functions

R9   define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease and original value problems, and simple interest including in financial mathematics

R10   solve problems involving direct and inverse proportion, including graphical and algebraic representations

R11   use compound units such as speed, rates of pay, unit pricing, <u>density and pressure</u>

R12   compare lengths, areas and volumes using ratio notation; <u>make links to similarity (including trigonometric ratios)</u> and scale factors

R13   <u>understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$</u> ; **construct and** <u>interpret equations that describe direct and inverse proportion</u>

R14   <u>interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion</u>

R15   **interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of average and instantaneous rate of change (gradients of chords and tangents) in numerical, algebraic and graphical contexts (this does not include calculus)**

R16   <u>set up, solve and interpret the answers in growth and decay problems, including compound interest</u> **and work with general iterative processes**

4. Geometry and measures

Properties and constructions

What students need to learn:

G1   use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description

G2   use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line

G3   apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles; understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)

G4   derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language

G5   use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)

G6   apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs

G7   identify, describe and construct congruent and similar shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement (including fractional **and negative** scale factors)

G8   **describe the changes and invariance achieved by combinations of rotations, reflections and translations**

G9   identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment

G10   **apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results**

G11   solve geometrical problems on coordinate axes

G12   identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres

G13   construct and interpret plans and elevations of 3D shapes

Mensuration and calculation

What students need to learn:

G14   use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)

G15   measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings

G16   know and apply formulae to calculate: area of triangles, parallelograms, trapezia; volume of cuboids and other right prisms (including cylinders)

G17   know the formulae: circumference of a circle = $2\pi r = \pi d$, area of a circle = πr^2; calculate: perimeters of 2D shapes, including circles; areas of circles and composite shapes; surface area and volume of spheres, pyramids, cones and composite solids

G18   calculate arc lengths, angles and areas of sectors of circles

G19   apply the concepts of congruence and similarity, including the relationships between lengths, **areas and volumes** in similar figures

G20   know the formulae for: Pythagoras' theorem $a^2 + b^2 = c^2$, and the trigonometric ratios, $\sin \theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$, $\cos \theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ and $\tan \theta = \dfrac{\text{opposite}}{\text{adjacent}}$; apply them to find angles and lengths in right-angled triangles **and, where possible, general triangles** in two **and three** dimensional figures

G21   know the exact values of $\sin \theta$ and $\cos \theta$ for $\theta = 0°$, 30°, 45°, 60° and 90°; know the exact value of $\tan \theta$ for $\theta = 0°$, 30°, 45° and 60°

G22   **know and apply the sine rule** $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$**, and cosine rule** $a^2 = b^2 + c^2 - 2bc \cos A$**, to find unknown lengths and angles**

G23   **know and apply Area =** $\dfrac{1}{2} ab \sin C$ **to calculate the area, sides or angles of any triangle**

Vectors

What students need to learn:

G24   describe translations as 2D vectors

G25   apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; **use vectors to construct geometric arguments and proofs**

5. Probability

What students need to learn:

P1   record, describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees

P2   apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments

P3   relate relative expected frequencies to theoretical probability, using appropriate language and the 0-1 probability scale

P4   apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one

P5   understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size

P6   enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams and tree diagrams

P7   construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities

P8   calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions

P9   **calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams**

6. Statistics

What students need to learn:

S1 infer properties of populations or distributions from a sample, while knowing the limitations of sampling

S2 interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time series data and know their appropriate use

S3 **construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use**

S4 interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:

- appropriate graphical representation involving discrete, continuous and grouped data, **including box plots**

- appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers, **quartiles and inter-quartile range**)

S5 apply statistics to describe a population

S6 use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends while knowing the dangers of so doing